

Strong coupling constant of negative parity octet baryons with light pseudoscalar mesons in light cone QCD sum rules

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Abstract

The strong coupling constants of the π and K mesons with negative parity octet baryons are estimated within the light cone QCD sum rules. It is observed that all strong coupling constants, similar to the case for the positive parity baryons, can be described in terms of three invariant functions, where two of them correspond to the well known F and D couplings in the $SU(3)_f$ symmetry, and the third function describes the $SU(3)_f$ symmetry violating effects. We compare our predictions on the strong coupling constants of pseudoscalar mesons of negative parity baryons with those corresponding to the strong coupling constants for the positive parity baryons.

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1 Introduction

The hadron–meson strong coupling constants play central role in the analysis of the existing experimental results of hadron physics. Among the strong coupling constants of baryons with mesons only nucleon–pion coupling constant is well determined from the experiments available at present. However, the situation for the K meson is not that simple. In reproducing the experimental results for the kaon–nucleon scattering and kaon photoproduction many phenomenologically unknown coupling constants are necessary (see [1]). Moreover, as far as negative parity baryons are concerned there exists limited experimental data. Understanding dynamics of the negative parity baryons requires formidable efforts, both from the theoretical and the experimental sides.

During the recent years considerable progress has been made in determination of the masses, magnetic moments of the negative parity baryons (see for example, [2–11]). Following these works, light cone QCD sum rules is also applied in studying the electromagnetic transition form factors for the $\gamma^*N \rightarrow N(1520)$ [12]. The next step in investigation of the properties of the negative parity baryons would be the analysis of their coupling constants with light pseudoscalar mesons. These coupling constants are the key parameters for studying the physics of negative parity baryons.

The present work is devoted to the study of the strong coupling constants of negative parity octet baryons (O^*) with light pseudoscalar mesons within the light cone QCD sum rules. The work is arranged as follows. In Section 2 the relevant sum rules for the strong coupling constants of the negative parity baryons with the light pseudoscalar mesons are derived. In Section 3 the numerical analysis of the sum rules obtained for the strong coupling constant $g_{O^*O^*\mathcal{P}}$ is performed.

2 Light cone sum rules for the $g_{O^*O^*\mathcal{P}}$ coupling constant

In order to determine $g_{O^*O^*\mathcal{P}}$ coupling constant we start by considering the following correlation function,

$$\Pi(p, q) = i \int d^4x e^{ipx} \left\langle \mathcal{P}(q) \left| T \left\{ \eta_{B_2}(x) \bar{\eta}_{B_1}(0) \right\} \right| 0 \right\rangle, \quad (1)$$

where $B_2(B_1)$ are the final (initial) baryons, η is the interpolating current of the corresponding baryon, q is the momentum of the pseudoscalar meson. Following the strategy of the QCD sum rules, we must first compute the correlation function (1) in two different kinematical domains. If the initial and final baryons are close to the mass shell of the corresponding baryons, i.e., $p^2 \simeq m_{B_2}^2$ and $(p+q)^2 \simeq m_{B_1}^2$ the hadronic part of the correlation function will be dominated by the processes $O^* \rightarrow O^*\mathcal{P}$ and $O^* \rightarrow O\mathcal{P}$, i.e., octet baryons of both parities contribute to the correlation function.

It should be noted here that in the case of nucleons, in addition to the $N(938)$ state the next positive parity $N(1440)$ state also gives contribution to the correlation function, while such states are absent in all other members of octet baryons. This fact makes the calculation of the nucleon coupling constant $g_{N^*N^*\pi}$ more challenging, whose technical details is discussed in [13].

The general form of the octet baryon currents are,

$$\begin{aligned}
\eta_{\Sigma^+} &= 2\varepsilon^{abc} \sum_{\ell=1}^2 (u^{aT} C A_1^\ell s^b) A_2^\ell u^c, \\
\eta_{\Sigma^+} &= \eta_{\Sigma^+}(u \rightarrow d), \\
\eta_{\Sigma^0} &= \sqrt{2}\varepsilon^{abc} \sum_{\ell=1}^2 \{ (u^{aT} C A_1^\ell s^b) A_2^\ell d^c + (d^{aT} C A_1^\ell s^b) A_2^\ell u^c \}, \\
\eta_{\Sigma^0} &= \eta_{\Sigma^+}(u \rightarrow s), \\
\eta_{\Xi^-} &= \eta_{\Sigma^-}(d \rightarrow s).
\end{aligned} \tag{2}$$

Here C is the charge conjugation operator, $A_1^1 = I$, $A_1^2 = A_2^1 = \gamma_5$, and $A_2^2 = \beta$ where β is an arbitrary auxiliary parameter. It is shown in [14] that the current of Λ baryon can be obtained from the Σ^0 current with the help of following relations,

$$\begin{aligned}
2\eta_{\Sigma^0}(d \leftrightarrow s) + \eta_{\Sigma^0} &= -\sqrt{3}\eta_\Lambda \quad \text{or,} \\
2\eta_{\Sigma^0}(u \leftrightarrow s) - \eta_{\Sigma^0} &= -\sqrt{3}\eta_\Lambda.
\end{aligned} \tag{3}$$

Note that these currents interact with both positive and negative parity baryons.

In order to find the expression of the correlation function from the hadronic part the correlation function (1) needs to be saturated with the hadronic states as follows,

$$\Pi(p, q) = \sum_{i,j} \frac{\langle 0 | \eta_2 | B_2^i(p) \rangle \langle B_2^i(p) \mathcal{P}(q) | B_1^j(p+q) \rangle \langle B_1^j(p+q) | \bar{\eta}_1 | 0 \rangle}{(p_i^2 - m_{2i}^2)[(p+q)^2 - m_{1j}^2]}, \tag{4}$$

where the summation over positive and negative parity baryons are implemented by the subindices i and j . The matrix elements entering into Eq. (1) are determined as,

$$\begin{aligned}
\langle 0 | \eta | B^+(p, s) \rangle &= \lambda_+ u_+(p, s), \\
\langle 0 | \eta | B^-(p, s) \rangle &= \lambda_- \gamma_5 u_+(p, s), \\
\langle B_2(p)^+ \mathcal{P} | B_1^+(p+q) \rangle &= i g_{++} \bar{u}_+ \gamma_5 u_+, \\
\langle B_2(p)^- \mathcal{P} | B_1^-(p+q) \rangle &= i g_{--} \bar{u}_- \gamma_5 u_-, \\
\langle B_2(p)^+ \mathcal{P} | B_1^-(p+q) \rangle &= i g_{-+} \bar{u}_+ u_-, \\
\langle B_2(p)^- \mathcal{P} | B_1^+(p+q) \rangle &= i g_{+-} \bar{u}_- u_+,
\end{aligned} \tag{5}$$

Using these matrix elements and performing the summation over the spins of Dirac spinors, the hadronic part of the correlation function can be written as,

$$\begin{aligned}
\Pi &= i g_{++} \lambda_{1+} \lambda_{2+} \frac{(\not{p} + m_{2+}) \gamma_5 (\not{p} + \not{q} + m_{1+})}{[(p+q)^2 - m_{1+}^2](p^2 - m_{2+}^2)} \\
&\quad - i g_{--} \lambda_{1-} \lambda_{2-} \frac{\gamma_5 (\not{p} + m_{2-}) \gamma_5 (\not{p} + \not{q} + m_{1-}) \gamma_5}{[(p+q)^2 - m_{1-}^2](p^2 - m_{2-}^2)} \\
&\quad - i g_{-+} \lambda_{1-} \lambda_{2+} \frac{(\not{p} + m_{2+}) (\not{p} + \not{q} + m_{1-}) \gamma_5}{[(p+q)^2 - m_{1-}^2](p^2 - m_{2+}^2)} \\
&\quad + i g_{+-} \lambda_{1+} \lambda_{2-} \frac{\gamma_5 (\not{p} + m_{2-}) (\not{p} + \not{q} + m_{1+})}{[(p+q)^2 - m_{1+}^2](p^2 - m_{2-}^2)}.
\end{aligned} \tag{6}$$

Few words are in order about the contributions that are expected to come from single pole terms for each state. In principle the strong coupling constant $g_{\mathcal{P}O^*O^*}$ has q^2 momentum dependence since it contains pseudoscalar meson form factor, which is described by the single pole contributions. In the framework of the light cone QCD sum rules approach we have used, the pseudoscalar mesons are assumed on the mass shell, i.e., $q^2 = m_{\mathcal{P}}^2$, and therefore the strong coupling constant $g_{\mathcal{P}O^*O^*}$ has no q^2 dependency. Under this condition the correlation function depends on two variables, namely p^2 and $(p+q)^2$, and have double poles with respect to these variables. In general single pole terms appear in subtraction procedure, which makes double dispersion integral finite.

In determination of the strong coupling constants of pseudoscalar mesons with negative parity octet baryons, the double Borel transformation is performed with respect to the variables $-p^2$ and $-(p+q)^2$, and the single pole terms that depend on only either one of these variables vanish.

In order to calculate the light pseudoscalar meson–negative parity hyperon coupling constant g_{--} , the coupling constant g_{++} between the positive–positive parity baryons, as well as the coupling constants g_{-+} and g_{+-} between the negative–positive and positive–negative parity hyperons, respectively, must be eliminated from the four coupled linear equations. For this purpose we need to calculate the correlation function from the QCD side.

Before proceeding further, following the approach given in [15] and [16], we first derive the corresponding relations among the correlation functions describing the coupling constants of the negative parity octet baryons with light pseudoscalar mesons. It is shown in these studies that all correlation functions can be expressed in terms of three independent functions, where two of them correspond to the $SU(3)_f$ symmetry limit and the third function takes $SU(3)_f$ violation into account. This is quite an exciting result since the coupling constants of all pseudoscalar mesons with baryons are all expressed in terms of F and D constants in the $SU(3)_f$ symmetry. Another advantage of this approach is that the relations among the invariant functions are structure independent. It can be shown by a specific example how these three invariant functions appear in the approach we consider. Consider the invariant function

$$\Pi = g_{\pi^0 \bar{u}u} \Pi_1(u, d, s) + g_{\pi^0 \bar{d}d} \Pi'_1(u, d, s) + g_{\pi^0 \bar{s}s} \Pi_2(u, d, s), \quad (7)$$

for the $\Sigma^0 \rightarrow \Sigma^0 \pi^0$ transition, where Σ^0 is the common notation for the positive and negative parity baryons. The quark content of the π^0 meson is formally written as,

$$J = \sum_{q=u,d,s} g_{\pi^0 \bar{q}q} \bar{q} \gamma_5 q,$$

and for the π^0 meson $g_{\pi^0 \bar{u}u} = -g_{\pi^0 \bar{d}d} = 1/\sqrt{2}$ and $g_{\pi^0 \bar{s}s} = 0$. Since the interpolating current of Σ^0 is symmetric under the exchange $u \leftrightarrow d$, i.e., $\Pi'_1(u, d, s) = \Pi_1(d, u, s)$, so can write,

$$\Pi^{\Sigma^0 \rightarrow \Sigma^0 \pi^0} = \frac{1}{\sqrt{2}} \left[\Pi_1(u, d, s) - \Pi_1(d, u, s) \right], \quad (8)$$

and obviously this invariant function is equal to zero at $SU(2)_f$ limit. For the convenience we introduce the following formal notation,

$$\Pi_1(u, d, s) = \langle \bar{u}u | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle,$$

$$\Pi_2(u, d, s) = \langle \bar{s}s | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle . \quad (9)$$

Making the replacement $d \rightarrow u$ in Π_1 and using $\Sigma^0(d \rightarrow u) = -\sqrt{2}\Sigma^+$, one can easily show that,

$$\langle \bar{u}u | \Sigma^+ \bar{\Sigma}^+ | 0 \rangle = 2\Pi_1(u, u, s) ,$$

so that we can write

$$\Pi^{\Sigma^+ \rightarrow \Sigma^+ \pi^0} = \sqrt{2}\Pi_1(u, u, s) . \quad (10)$$

The invariant functions involving Λ baryon appear with the ones involving Σ^0 baryon (see Eq. (3)) and therefore it is impossible to write them using only Π_1 and Π_2 . Hence we need to introduce one more independent function in order to separate the contributions coming from Λ and Σ^0 which can be written as,

$$\Pi_3(u, d, s) = \Pi^{\Sigma^0 \rightarrow \Xi^- K^+} = -\langle u\bar{s} | \Xi^- \Sigma^0 | 0 \rangle .$$

The relations among invariant functions in the isospin symmetry limit are given in the Appendix (see also [15]).

We now proceed to calculate the correlation functions from the QCD side. As already mentioned, all considered correlation functions can be written in terms of three invariant functions, which can be determined from the $\Sigma^0 \rightarrow \Sigma^0 \pi^0$ and $\Sigma^0 \rightarrow \Xi^- K^+$ transitions. Therefore in order to find the relations among correlation functions it is enough to calculate correlation functions responsible for these decays. Theoretical part of the correlation function describing the above transitions can be calculated at the deep Euclidian region $p^2 \ll 0$ and $(p+q)^2 \ll 0$ using the OPE over twists. In these calculations there appear matrix elements of nonlocal operators between vacuum and pseudoscalar meson states, such as, $\langle \mathcal{P}(q) | \bar{q}_1(x_1) \Gamma q_2(x_2) | 0 \rangle$ or $\langle \mathcal{P}(q) | \bar{q}_1(x_1) \Gamma G q_2(x_2) | 0 \rangle$, where Γ is the gluon field strength tensor. It should be noted here that contributions coming from four-particle nonlocal operators like $\bar{q} G G q$, $\bar{q} q \bar{q} q$ are all neglected. Formally, neglecting the contributions of these operators can be justified on the basis of an expansion in the conformal spin method [17]. Therefore we restrict ourselves to the contributions of the two- and three-particle pseudoscalar meson DAs up to twist four. In these calculations we also need the light quark propagator in the presence of background gluon field whose expression is given in [18],

$$S_q(x) = \frac{i\not{x}}{2\pi^2 x^4} [x, 0] - \frac{ig_s}{16\pi^2 x^2} \int_0^1 du [x, ux] \left[\bar{u} \not{x} \sigma_{\alpha\beta} + u \sigma_{\alpha\beta} \right] G^{\alpha\beta} [ux, 0] + \dots$$

where the aberration $[x, y]$ means,

$$[x, y] = \text{Pexp} \left[i \int dt (x - y)_\mu g B^\mu (tx - \bar{t}y) \right] ,$$

for the path ordered exponent, and B is the gluon field. The calculations are performed in the Fock-Schwinger gauge, i.e., $x_\mu B^\mu = 0$. As the result the massless quark propagator can be written as,

$$S_q(x) = \frac{i\not{x}}{2\pi^2 x^4} - ig_s \int_0^1 du \left[\frac{\bar{u} \not{x} \sigma_{\alpha\beta} G^{\alpha\beta}(ux)}{16\pi^2 x^2} - \frac{ux_\alpha \gamma_\beta G^{\alpha\beta}(ux)}{4\pi^2 x^2} \right] .$$

If the mass of the light quark is taken into account this propagator is modified as,

$$S_q(x) = \frac{i\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - ig_s \int_0^1 du \left[\frac{\bar{u}\not{x}\sigma_{\alpha\beta}G^{\alpha\beta}(ux)}{16\pi^2 x^2} - \frac{ux_\alpha\gamma_\beta G^{\alpha\beta}(ux)}{4\pi^2 x^2} \right. \\ \left. - \frac{im_q x^2}{32}\sigma_{\alpha\beta}G^{\alpha\beta}(ux) \left(\ln \frac{-x^2\Lambda^2}{4} + 2\gamma_E \right) \right],$$

where $\gamma_E \approx 0.577$ is the Euler constant, and Λ is the scale that separates the short and long distance domains. We choose this parameter as the factorization scale that has the value $\Lambda = 1 \text{ GeV}$ (see for example [19]).

The matrix elements of the nonlocal operators between one pseudoscalar and vacuum states are defined in terms of the meson distribution amplitudes as, [20–22]

$$\begin{aligned} \langle \mathcal{P}(q) | \bar{q}_1(x) \gamma_\mu \gamma_5 q_1(0) | 0 \rangle &= -if_{\mathcal{P}} q_\mu \int_0^1 du e^{i\bar{u}qx} \left(\varphi_{\mathcal{P}}(u) + \frac{1}{16} m_{\mathcal{P}}^2 x^2 \mathbb{A}(u) \right) \\ &\quad - \frac{i}{2} f_{\mathcal{P}} m_{\mathcal{P}}^2 \frac{x_\mu}{qx} \int_0^1 du e^{i\bar{u}qx} \mathbb{B}(u), \\ \langle \mathcal{P}(q) | \bar{q}_1(x) i\gamma_5 q_2(0) | 0 \rangle &= \mu_{\mathcal{P}} \int_0^1 du e^{i\bar{u}qx} \phi_P(u), \\ \langle \mathcal{P}(q) | \bar{q}_1(x) \sigma_{\alpha\beta} \gamma_5 q_2(0) | 0 \rangle &= \frac{i}{6} \mu_{\mathcal{P}} (1 - \tilde{\mu}_{\mathcal{P}}^2) (q_\alpha x_\beta - q_\beta x_\alpha) \int_0^1 du e^{i\bar{u}qx} \phi_\sigma(u), \\ \langle \mathcal{P}(q) | \bar{q}_1(x) \sigma_{\mu\nu} \gamma_5 g_s G_{\alpha\beta}(vx) q_2(0) | 0 \rangle &= i\mu_{\mathcal{P}} \left[q_\alpha q_\mu \left(g_{\nu\beta} - \frac{1}{qx} (q_\nu x_\beta + q_\beta x_\nu) \right) \right. \\ &\quad - q_\alpha q_\nu \left(g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) \\ &\quad - q_\beta q_\mu \left(g_{\nu\alpha} - \frac{1}{qx} (q_\nu x_\alpha + q_\alpha x_\nu) \right) \\ &\quad \left. + q_\beta q_\nu \left(g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) \right] \\ &\quad \times \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}(\alpha_i), \\ \langle \mathcal{P}(q) | \bar{q}_1(x) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(vx) q_2(0) | 0 \rangle &= q_\mu (q_\alpha x_\beta - q_\beta x_\alpha) \frac{1}{qx} f_{\mathcal{P}} m_{\mathcal{P}}^2 \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}_{\parallel}(\alpha_i) \\ &\quad + \left[q_\beta \left(g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) \right. \\ &\quad \left. - q_\alpha \left(g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) \right] f_{\mathcal{P}} m_{\mathcal{P}}^2 \\ &\quad \times \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}_{\perp}(\alpha_i), \\ \langle \mathcal{P}(q) | \bar{q}_1(x) \gamma_\mu i g_s G_{\alpha\beta}(vx) q_2(0) | 0 \rangle &= q_\mu (q_\alpha x_\beta - q_\beta x_\alpha) \frac{1}{qx} f_{\mathcal{P}} m_{\mathcal{P}}^2 \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}_{\parallel}(\alpha_i) \\ &\quad + \left[q_\beta \left(g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) \right. \end{aligned}$$

$$\begin{aligned}
& -q_\alpha \left(g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) \Big] f_{\mathcal{P}} m_{\mathcal{P}}^2 \\
& \times \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}_\perp(\alpha_i), \tag{11}
\end{aligned}$$

where

$$\mu_{\mathcal{P}} = f_{\mathcal{P}} \frac{m_{\mathcal{P}}^2}{m_{q_1} + m_{q_2}}, \quad \tilde{\mu}_{\mathcal{P}} = \frac{m_{q_1} + m_{q_2}}{m_{\mathcal{P}}},$$

and $\mathcal{D}\alpha = d\alpha_{\bar{q}} d\alpha_q d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g)$ is the measure. Here $\varphi_{\mathcal{P}}(u)$ is the leading twist-two, $\phi_P(u)$, $\phi_\sigma(u)$, $\mathcal{T}(\alpha_i)$ are the twist-three, and $\mathbb{A}(u)$, $\mathbb{B}(u)$, $\mathcal{A}_\perp(\alpha_i)$, $\mathcal{A}_\parallel(\alpha_i)$, $\mathcal{V}_\perp(\alpha_i)$ and $\mathcal{V}_\parallel(\alpha_i)$ are the twist-four DAs, respectively. The explicit forms of these matrix elements are given in the following section.

In order to proceed with the calculation of the strong coupling constant between negative parity baryons with light pseudoscalar mesons, we need to solve Eq. (5) for g_{--} . We see from this equation that four linearly independent equations are needed for determination of the strong coupling constant g_{--} , and hence we choose four different Lorentz structures, $\not{p}\not{q}\gamma_5$, $\not{p}\gamma_5$, $\not{q}\gamma_5$ and γ_5 which correspond to the invariant functions $\Pi_i^{(1)}$, $\Pi_i^{(2)}$, $\Pi_i^{(3)}$, and $\Pi_i^{(4)}$ respectively. Here the subindex i runs over the invariant functions Π_1 , Π_2 , Π_3 as are introduced earlier.

In order to find sum rules for the strong coupling constant of the negative parity octet baryons with pseudoscalar mesons, we should match both representations of the correlation function from the hadronic and QCD sides. Performing the double Borel transformation over the variables $-p^2$ and $-(p+q)^2$ the higher states and continuum contributions are suppressed. The contributions are calculated by using the quark-hadron duality, i.e., above some threshold, the hadronic spectral density is equal to the spectral density calculated in terms of quark-gluon degrees of freedom. Following the Borel transformation, the continuum subtraction procedure is applied whose details are given in [21]. Setting $M_1^2 = M_2^2 = 2M^2$ and $u_0 = 1/2$ the continuum subtraction procedure can be done using the relation,

$$(M^2)^n = \frac{1}{\Gamma(n)} \int_0^{s_0} ds e^{-s/M^2} s^{n-1}.$$

The continuum subtraction is not performed for the higher twist terms due to the fact that their contributions are known to be small (for more details see [21]).

Having implemented the Borel transformation and continuum subtraction procedures we obtain four equations, which correspond to the transitions between positive-positive, negative-negative, positive-negative and negative-positive parity baryons.

$$\begin{aligned}
& -A + B - C + D = \Pi_i^{B(1)}, \\
& -m_{2+}A - m_{2-}B - m_{2+}C - m_{2-}D = \Pi_i^{B(2)}, \\
& (m_{1+} - m_{2-})A + (m_{1-} - m_{2-})B - (m_{1-} + m_{2+})C - (m_{1+} + m_{2-})D = \Pi_i^{B(3)}, \\
& m_{2+}(m_{1+} - m_{2+})A - m_{2-}(m_{1-} - m_{2-})B - m_{2+}(m_{1-} + m_{2+})C \\
& + m_{2-}(m_{1+} + m_{2-})D = \Pi_i^{B(4)}, \tag{12}
\end{aligned}$$

where

$$\begin{aligned} A &= g_{++}\lambda_{1+}\lambda_{2+}e^{-(m_{1+}^2+m_{2+}^2)/2M^2}, \\ B &= g_{--}\lambda_{1-}\lambda_{2-}e^{-(m_{1-}^2+m_{2-}^2)/2M^2}, \\ C &= g_{-+}\lambda_{1-}\lambda_{2+}e^{-(m_{1-}^2+m_{2+}^2)/2M^2}, \\ D &= g_{+-}\lambda_{1+}\lambda_{2-}e^{-(m_{1+}^2+m_{2-}^2)/2M^2}. \end{aligned}$$

In order to obtain the $g_{\mathcal{P}O^*O^*}$ coupling constant we need to determine the function B only. Solving these four equations the strong coupling constant is found to be,

$$\begin{aligned} g_{--} &= \frac{e^{(m_{1-}^2+m_{2-}^2)/2M^2+m_{\mathcal{P}}^2/4M^2}}{\lambda_{1-}\lambda_{2-}(m_{1-}+m_{1+})(m_{2-}+m_{2+})} \left\{ (m_{1+}+m_{2-})m_2^+\Pi_i^{B(1)} + m_{2+}\Pi_i^{B(2)} \right. \\ &\quad \left. - (m_{1+}+m_{2-})\Pi_i^{B(3)} - \Pi_i^{B(4)} \right\}, \end{aligned} \quad (13)$$

where $\Pi_i^{B(j)}$ correspond to the invariant functions $\Pi_i^{(j)}$ after the Borel transformation with respect to the variables $-p^2$ and $-(p+q)^2$. The explicit expressions of the invariant functions $\Pi^{B(i)}$ ($i = 1, 2, 3, 4$) for any considered strong coupling constant between negative parity baryons with light pseudoscalar mesons are quite lengthy, and hence we do not present their explicit forms.

It follows from Eq. (13) that to able to estimate of the strong coupling constants of negative parity octet baryons with light pseudoscalar mesons, residues of negative parity octet baryons are needed. These residues are calculated in [11, 12]

3 Numerical analysis

This section is devoted to the numerical analysis of the sum rules obtained in Section 3 for the strong coupling constants of the light pseudoscalar mesons with negative parity octet baryons. The sum rules of the aforementioned coupling constants contain the DAs of the pseudoscalar mesons as the input parameters, and their explicit forms are given below [20–22].

$$\begin{aligned} \varphi_{\mathcal{P}}(u) &= 6u\bar{u} \left[1 + a_1^{\mathcal{P}}C_1(2u-1) + a_2^{\mathcal{P}}C_2^{3/2}(2u-1) \right], \\ \mathcal{T}(\alpha_i) &= 360\eta_3\alpha_{\bar{q}}\alpha_q\alpha_g^2 \left[1 + w_3\frac{1}{2}(7\alpha_g-3) \right], \\ \phi_P(u) &= 1 + \left[30\eta_3 - \frac{5}{2}\frac{1}{\mu_{\mathcal{P}}^2} \right] C_2^{1/2}(2u-1), \\ &\quad + \left(-3\eta_3w_3 - \frac{27}{20}\frac{1}{\mu_{\mathcal{P}}^2} - \frac{81}{10}\frac{1}{\mu_{\mathcal{P}}^2}a_2^{\mathcal{P}} \right) C_4^{1/2}(2u-1), \\ \phi_{\sigma}(u) &= 6u\bar{u} \left[1 + \left(5\eta_3 - \frac{1}{2}\eta_3w_3 - \frac{7}{20}\mu_{\mathcal{P}}^2 - \frac{3}{5}\mu_{\mathcal{P}}^2a_2^{\mathcal{P}} \right) C_2^{3/2}(2u-1) \right], \\ \mathcal{V}_{\parallel}(\alpha_i) &= 120\alpha_q\alpha_{\bar{q}}\alpha_g(v_{00}+v_{10}(3\alpha_g-1)), \end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{\parallel}(\alpha_i) &= 120\alpha_q\alpha_{\bar{q}}\alpha_g(0 + a_{10}(\alpha_q - \alpha_{\bar{q}})) , \\
\mathcal{V}_{\perp}(\alpha_i) &= -30\alpha_g^2 \left[h_{00}(1 - \alpha_g) + h_{01}(\alpha_g(1 - \alpha_g) - 6\alpha_q\alpha_{\bar{q}}) + h_{10}(\alpha_g(1 - \alpha_g) - \frac{3}{2}(\alpha_{\bar{q}}^2 + \alpha_q^2)) \right] , \\
\mathcal{A}_{\perp}(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q) \left[h_{00} + h_{01}\alpha_g + \frac{1}{2}h_{10}(5\alpha_g - 3) \right] , \\
B(u) &= g_{\mathcal{P}}(u) - \varphi_{\mathcal{P}}(u) , \\
g_{\mathcal{P}}(u) &= g_0C_0^{1/2}(2u - 1) + g_2C_2^{1/2}(2u - 1) + g_4C_4^{1/2}(2u - 1) , \\
\mathbb{A}(u) &= 6u\bar{u} \left[\frac{16}{15} + \frac{24}{35}a_2^{\mathcal{P}} + 20\eta_3 + \frac{20}{9}\eta_4 + \left(-\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\eta_3w_3 - \frac{10}{27}\eta_4 \right) C_2^{3/2}(2u - 1) \right. \\
&\quad \left. + \left(-\frac{11}{210}a_2^{\mathcal{P}} - \frac{4}{135}\eta_3w_3 \right) C_4^{3/2}(2u - 1) \right] , \\
&\quad + \left(-\frac{18}{5}a_2^{\mathcal{P}} + 21\eta_4w_4 \right) [2u^3(10 - 15u + 6u^2) \ln u \\
&\quad + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2) \ln \bar{u} + u\bar{u}(2 + 13u\bar{u})] . \tag{14}
\end{aligned}$$

Here $C_n^k(x)$ are the Gegenbauer polynomials, and

$$\begin{aligned}
h_{00} &= v_{00} = -\frac{1}{3}\eta_4 , \\
a_{10} &= \frac{21}{8}\eta_4w_4 - \frac{9}{20}a_2^{\mathcal{P}} , \\
v_{10} &= \frac{21}{8}\eta_4w_4 , \\
h_{01} &= \frac{7}{4}\eta_4w_4 - \frac{3}{20}a_2^{\mathcal{P}} , \\
h_{10} &= \frac{7}{4}\eta_4w_4 + \frac{3}{20}a_2^{\mathcal{P}} , \\
g_0 &= 1 , \\
g_2 &= 1 + \frac{18}{7}a_2^{\mathcal{P}} + 60\eta_3 + \frac{20}{3}\eta_4 , \\
g_4 &= -\frac{9}{28}a_2^{\mathcal{P}} - 6\eta_3w_3 . \tag{15}
\end{aligned}$$

The values of the parameters $a_1^{\mathcal{P}}$, $a_2^{\mathcal{P}}$, η_3 , η_4 , w_3 , and w_4 entering Eq. (14) are listed in Table (1) for the pseudoscalar π , K and η mesons.

In addition to the DAs, these coupling constants contain also additional parameters such as quark condensates and magnetic susceptibility of quarks. In the present analysis we use $\langle \bar{u}u \rangle|_{\mu=1 \text{ GeV}} = \langle \bar{d}d \rangle|_{\mu=1 \text{ GeV}} = -(0.243)^3 \text{ GeV}^3$ [23], $\langle \bar{s}s \rangle|_{\mu=1 \text{ GeV}} = 0.8\langle \bar{u}u \rangle|_{\mu=1 \text{ GeV}}$, $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ [24]. Magnetic susceptibility of the quarks are determined in the framework of the QCD sum rules in [25–27] and in our work we use $\chi(1 \text{ GeV}) = -2.85 \text{ GeV}^2$ predicted in [27].

Besides these input parameters the sum rules contain three auxiliary parameters: Borel mass square M^2 , continuum threshold s_0 and the arbitrary parameter β in expressions of the interpolating currents. For a reliable determination of the strong coupling constants of $J^P = \frac{1}{2}^-$ octet baryons with pseudoscalar meson it is necessary to find such regions of

	π	K
$a_1^{\mathcal{P}}$	0	0.050
$a_2^{\mathcal{P}}$ (set-1)	0.11	0.15
$a_2^{\mathcal{P}}$ (set-2)	0.25	0.27
η_3	0.015	0.015
η_4	10	0.6
w_3	-3	-3
w_4	0.2	0.2

Table 1: Parameters of the wave function calculated at the renormalization scale $\mu = 1 \text{ GeV}$

these parameters where coupling constant exhibits good stability to the variation of them. Our analysis shows that the working region of M^2 in the present work coincides practically with the one determined for the magnetic moments of negative parity baryons in [12], as are given below,

$$\begin{aligned}
1.5 \text{ GeV}^2 &\leq M^2 \leq 3.0 \text{ GeV}^2, \text{ for } p^* \text{ and } n^*, \\
1.8 \text{ GeV}^2 &\leq M^2 \leq 3.5 \text{ GeV}^2, \text{ for } \Lambda^*, \Sigma^* \text{ and } \Xi^*.
\end{aligned}$$

The continuum threshold s_0 is related to the first excited states. The difference $\sqrt{s_0} - m_{\text{ground}}$ is the energy needed to transfer the baryon to its first excited state. Analysis of all existing sum rules approaches shows that this difference usually varies in the region from 0.3 GeV to 0.8 GeV , i.e., $m_{\text{ground}} + 0.3 \text{ GeV} \leq \sqrt{s_0} \leq m_{\text{ground}} + 0.8 \text{ GeV}$, and in our analysis we chose the average value $m_{\text{ground}} + 0.5 \text{ GeV}$.

After deciding on the working regions of M^2 and s_0 , our next and final goal is to find the region of the auxiliary parameter β where the results are independent with respect to its variation.

As an example in Figs. 1 and 2 we present the dependence of $g_{\Sigma^{0*}\Lambda^{0*}\pi^0}$ coupling constant on M^2 at four fixed values of β , and at $s_0 = 4.0 \text{ GeV}^2$ and $s_0 = 4.5 \text{ GeV}^2$, respectively. We see from these figures that the results are rather stable with respect to the variation in M^2 and β .

In Figs. 3 and 4 we depict the dependence of the $g_{\Sigma^{0*}\Lambda^{0*}\pi^0}$ on $\cos \theta$ (where $\beta = \tan \theta$) at three fixed values of M^2 , and at two fixed values of the continuum threshold $s_0 = 4.0 \text{ GeV}^2$ and 4.5 GeV^2 , respectively. We observe from these figures that when $\cos \theta$ varies in the interval $-1.00 \leq \cos \theta \leq -0.85$ the coupling constant seems to be independent on the parameter β and we get $g_{\Sigma^{0*}\Lambda^{0*}\pi^0} = 5.0 \pm 0.5$.

We perform similar analysis for the other coupling constants whose results are all summarized in Table 1.

For completeness, in Table 2 we also present the modular values of the strong coupling constants of the light pseudoscalar mesons with positive parity octet baryons.

From the results given in Table 1 we deduce the following conclusions:

	Negative parity baryons	Positive parity baryons
$\Lambda \rightarrow \Sigma^+ \pi^-$	4 ± 1	10 ± 3
$\Lambda \rightarrow \Xi^0 K^0$	4 ± 2	4.5 ± 2.0
$\Sigma^0 \rightarrow n K^0$	3 ± 1	4 ± 3
$\Sigma^0 \rightarrow \Lambda \pi^0$	5.0 ± 0.5	11 ± 3
$\Sigma^0 \rightarrow \Xi^0 K^0$	10 ± 2	13 ± 2
$\Sigma^- \rightarrow n K^-$	3 ± 1	5 ± 3
$\Sigma^+ \rightarrow \Lambda \pi^+$	6 ± 2	10.0 ± 3.5
$\Sigma^+ \rightarrow \Sigma^0 \pi^+$	8 ± 3	9 ± 2
$\Xi^0 \rightarrow \Lambda K^0$	4.0 ± 0.5	4.5 ± 1.0
$\Xi^0 \rightarrow \Sigma^0 K^0$	9 ± 1	12.5 ± 3.0
$\Xi^0 \rightarrow \Sigma^+ K^-$	14 ± 3	18 ± 4
$\Xi^0 \rightarrow \Xi^0 \pi^0$	4 ± 1	10 ± 2

Table 2: The strong coupling constants of negative and positive parity octet baryons with pseudoscalar mesons.

- In many cases the results obtained by using the general current differ considerably from the predictions of the Ioffe current ($\beta = -1$). This difference can be attributed to the fact that the coupling constant predicted by the Ioffe current lies outside the stability region of β . As an example the coupling constant is $g_{\Sigma^{0*}\Lambda^{0*}\pi^0} = 25$ for the Ioffe current ($\beta = -1$), while it is $g_{\Sigma^{0*}\Lambda^{0*}\pi^0} = 5.0 \pm 0.5$ for the general current.
- The values of the strong coupling constants for the negative and positive parity baryons are close to each other in many cases. Considerable difference occurs for the $\Sigma^{+*} \rightarrow \Lambda^{0*}\pi^+$, $\Sigma^{0*} \rightarrow \Lambda^{0*}\pi^0$, $\Lambda^{0*} \rightarrow \Sigma^{+*}\pi^-$, and $\Xi^{0*} \rightarrow \Xi^{0*}\pi^0$ channels.

Finally it should be emphasized here that our prediction on the coupling constant for the $N^* \rightarrow N^*\pi^0$ channel is approximately 50% larger than the one calculated in 3-point QCD sum rules approach [28].

In conclusion, the strong coupling constants of the light pseudoscalar mesons with the negative parity octet baryons are calculated in framework of the light cone QCD sum rules. We observe that all coupling constants can be described only in terms of three invariant functions, where two of them correspond to the well known F and D couplings in the $SU(3)_f$ symmetry, and the third one describes the $SU(3)_f$ violating effects. We present our predictions of the strong coupling constants of $J^P = \frac{1}{2}^-$ octet baryons and compare them with the coupling constants of the corresponding positive parity baryons.

Appendix

In this appendix we present the relations among the correlation functions in the isospin symmetry limit. These relations hold for the positive and negative parity baryons (see also [15]).

- Correlation functions involving the pions:

$$\begin{aligned}
\Pi^{\Sigma^0 \rightarrow \Sigma^0 \pi} &= \Pi^{\Lambda \rightarrow \Lambda \pi} = 0 \\
\sqrt{2}\Pi_1(q, q, s) &= \Pi^{\Sigma^+ \rightarrow \Sigma^+ \pi} = -\Pi^{\Sigma^- \rightarrow \Sigma^- \pi} = -\Pi^{\Sigma^0 \rightarrow \Sigma^+ \pi} \\
&= \Pi^{\Sigma^- \rightarrow \Sigma^0 \pi} = -\Pi^{\Sigma^+ \rightarrow \Sigma^0 \pi} = \Pi^{\Sigma^0 \rightarrow \Sigma^- \pi} \\
\Pi^{\Xi^0 \rightarrow \Xi^0 \pi} &= \frac{1}{\sqrt{2}}\Pi_2(s, s, q) = -\Pi^{\Xi^- \rightarrow \Xi^- \pi} = -\frac{1}{\sqrt{2}}\Pi^{\Xi^- \rightarrow \Xi^0 \pi} = -\frac{1}{\sqrt{2}}\Pi^{\Xi^0 \rightarrow \Xi^- \pi} \\
\Pi^{p \rightarrow p \pi} &= -\Pi^{n \rightarrow n \pi} = \sqrt{2}\Pi_1(q, q, q) - \frac{1}{\sqrt{2}}\Pi_2(q, q, q) \\
\Pi^{\Lambda \rightarrow \Sigma^+ \pi} &= \Pi^{\Lambda \rightarrow \Sigma^- \pi} = -\frac{1}{\sqrt{3}} \left[2\Pi_3(q, s, q) + \sqrt{2}\Pi_1(q, q, s) \right] \\
\Pi^{\Sigma^+ \rightarrow \Lambda \pi} &= \Pi^{\Sigma^- \rightarrow \Lambda \pi} = -\frac{1}{\sqrt{3}} \left[2\Pi_3(q, q, s) + \sqrt{2}\Pi_1(q, q, s) \right] \\
\Pi^{n \rightarrow p \pi} &= \Pi^{p \rightarrow n \pi} = -\sqrt{2}\Pi_3(q, q, q) \\
\Pi^{\Sigma^0 \rightarrow \Lambda \pi} + \Pi^{\Lambda \rightarrow \Sigma^0 \pi^0} &= \frac{4}{\sqrt{6}} [\Pi_1(q, s, q) - \Pi_2(s, q, q)]
\end{aligned}$$

- Correlation functions involving the kaons:

$$\begin{aligned}
\Pi^{n \rightarrow \Sigma^0 K} &= -\Pi^{p \rightarrow \Sigma^0 K} = \Pi_3(s, q, q) + \sqrt{2}\Pi_1(s, q, q) \\
\Pi^{p \rightarrow \Lambda K} &= \Pi^{n \rightarrow \Lambda K} = -\frac{1}{\sqrt{3}} \left[\sqrt{2}\Pi_1(s, q, q) - \Pi_3(s, q, q) \right] \\
\Pi^{p \rightarrow \Sigma^+ K} &= \Pi^{n \rightarrow \Sigma^- K} = -\Pi_2(q, q, q) \\
-\Pi^{\Sigma^0 \rightarrow \Xi^- K} &= -\frac{1}{\sqrt{2}}\Pi^{\Sigma^+ \rightarrow \Xi^0 K} = \Pi^{\Sigma^0 \rightarrow \Xi^0 K} = -\frac{1}{\sqrt{2}}\Pi^{\Sigma^- \rightarrow \Xi^- K} = \Pi_3(q, q, s) \\
\Pi^{\Lambda \rightarrow \Xi^0 K} &= \Pi^{\Lambda \rightarrow \Xi^- K} = \frac{1}{\sqrt{3}} \left[2\sqrt{2}\Pi_1(q, s, q) + \Pi_3(q, q, s) \right] \\
\Pi^{\Sigma^0 \rightarrow n K} &= -\Pi^{\Sigma^0 \rightarrow p K} = \sqrt{2}\Pi_1(s, q, q) + \Pi_3(s, q, q) \\
-\Pi^{\Lambda \rightarrow p K} &= -\Pi^{\Lambda \rightarrow n K} = \frac{1}{\sqrt{3}} \left[\sqrt{2}\Pi_1(s, q, q) - \Pi_3(s, q, q) \right] \\
\Pi^{\Sigma^- \rightarrow n K} &= \Pi^{\Sigma^+ \rightarrow p K} = -\Pi_2(q, q, s) \\
-\Pi^{\Xi^0 \rightarrow \Sigma^+ K} &= -\Pi^{\Xi^- \rightarrow \Sigma^- K} = \sqrt{2}\Pi_3(s, s, q) \\
-\Pi^{\Xi^- \rightarrow \Sigma^0 K} &= \Pi^{\Xi^0 \rightarrow \Sigma^0 K} = \Pi_3(q, s, q) \\
\Pi^{\Xi^- \rightarrow \Lambda K} &= \Pi^{\Xi^0 \rightarrow \Lambda K} = \frac{1}{\sqrt{3}} \left[2\sqrt{2}\Pi_1(q, s, q) + \Pi_3(q, q, s) \right]
\end{aligned}$$

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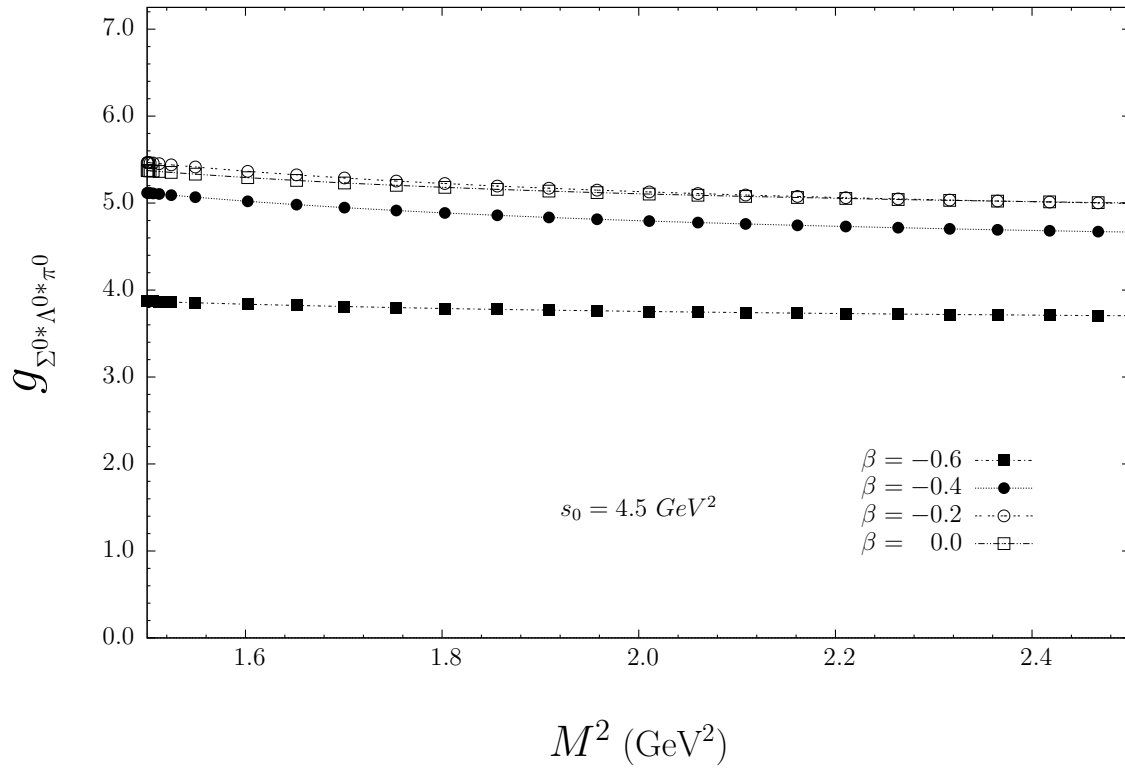
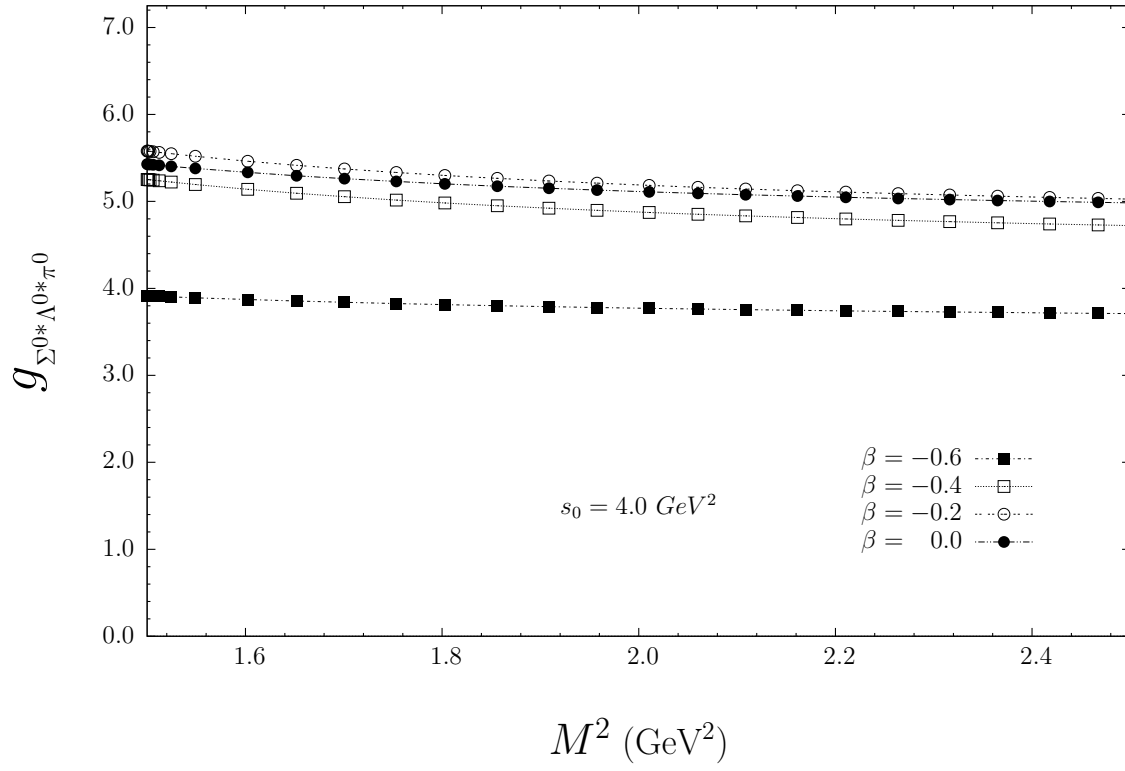
Figure captions

Fig. (1) The dependence of the strong coupling constant $g_{\Sigma^0\Lambda^0\pi^0}$ on the Borel parameter M^2 , at the fixed value of the continuum threshold $s_0 = 4.0 \text{ GeV}^2$, and several fixed values of the auxiliary parameter β .

Fig. (2) The same as Fig. (1), but at the fixed value of the continuum threshold $s_0 = 4.5 \text{ GeV}^2$.

Fig. (3) The dependence of the strong coupling constant $g_{\Sigma^0\Lambda^0\pi^0}$ on $\cos\theta$, at the fixed value of the continuum threshold $s_0 = 4.0 \text{ GeV}^2$, at several fixed values of M^2 .

Fig. (4) The same as Fig. (3), but at the fixed value of the continuum threshold $s_0 = 4.5 \text{ GeV}^2$.



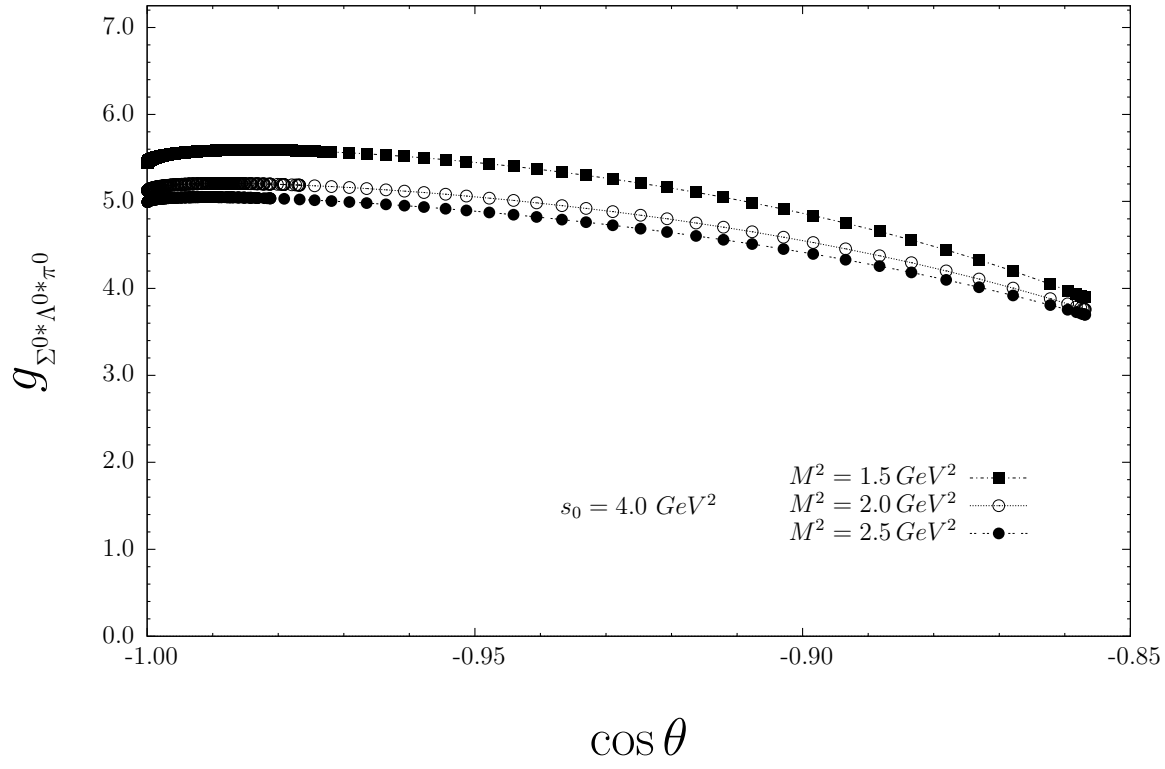


Figure 3:

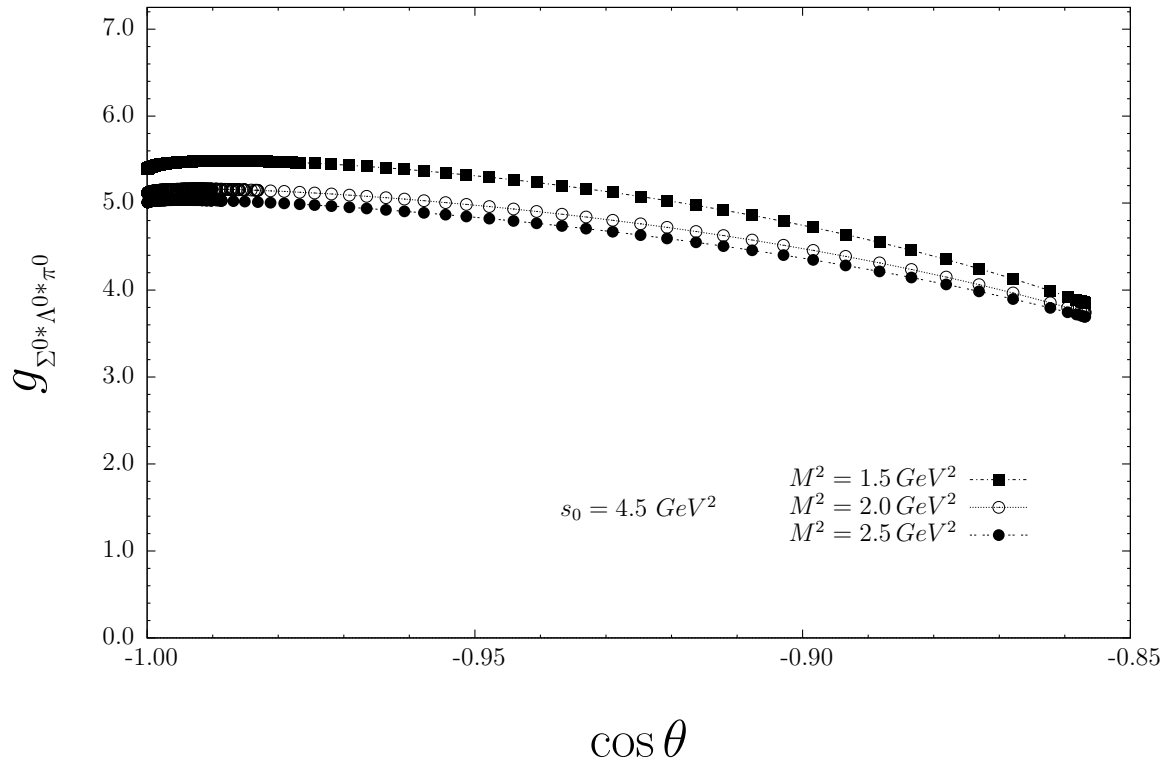


Figure 4: